

Naked Singularity and Gauss-Bonnet Term in Brane World Scenarios

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Abstract

We add a Gauss-Bonnet term to the Einstein-Hilbert action and study the recent proposal to solve the cosmological constant problem. We also consider the possibility of adding a dilaton potential to the action. In the absence of supersymmetry, we obtain first order Bogomol'nyi equation as a solution-generating method in our scenario. When the coefficient of the Gauss-Bonnet term is positive, the dilaton potential is bounded below. Assuming a simple double-well potential, we find the dilaton field to be a kink in the fifth dimension.

I. INTRODUCTION

It was suggested long time ago [1] that the cosmological constant problem could be solved if one is willing to go to higher dimensional spacetime. Recently, a number of authors [2–5] have outlined possibilities for solving this long standing problem within the brane world scenario [6,7]. In [2,3], our 4-dimensional world is embedded in a 5-dimensional universe and remains flat in the presence of an arbitrary vacuum energy density V . The price one pays for this remarkable phenomenon is the appearance of naked singularities in the 5-dimensional universe.

In this note, we consider adding to the action higher derivative terms, in particular the Gauss-Bonnet term, and/or a potential for the dilaton field. A simple anti-de Sitter solution and inflationary solutions to the action with Gauss-Bonnet term were studied in [8,9]. Models with gravity coupled to scalars are considered in [10,11], and in the context of five dimensional supergravity as well [12]. Here we explore the question of how the solution of the cosmological constant problem might be modified by the addition of these terms.

To introduce the subject and to set the stage for our discussion, let us review the proposed solution reduced to its simplest version. We will follow [2] which we will refer to as KSS. The low energy effective action is taken to be

$$S = \int d^5x \sqrt{-G} \left[R - \frac{4}{3}(\nabla\varphi)^2 \right] + \int d^4x \sqrt{-g}(-V). \quad (1)$$

Gravity and a dilaton field φ live in the 5-dimensional world and is coupled to a thin 4-dimensional brane whose position is taken to be at $y = 0$. Here $M, N = 0, 1, 2, 3, 5$ and $\mu, \nu = 0, 1, 2, 3$. The metric $g_{\mu\nu} = \delta_\mu^M \delta_\nu^N G_{MN}(y = 0)$ is the 4-dimensional metric on the brane. (We use the convention in which $R_{NPQ}^M = +\Gamma_{NQ,P}^M \cdots$, $R_{MN} = R_{MPN}^P$, $R = R_M^M$, and the signature $(-, +, +, +, +)$.)

Einstein's equations read

$$R_{MN} - \frac{1}{2}G_{MN}R - \frac{4}{3} \left[\nabla_M \varphi \nabla_N \varphi - \frac{1}{2}G_{MN}(\nabla\varphi)^2 \right] + \frac{1}{2}\sqrt{\frac{g}{G}} V g_{\mu\nu} \delta_M^\mu \delta_N^\nu \delta(y) = 0 \quad (2)$$

with g and G the determinant of $g_{\mu\nu}$ and G_{MN} respectively. The equation of motion for the dilaton field is given by

$$\nabla^2\varphi = 0. \quad (3)$$

A theorist living in the 4-dimensional brane world would notice that the action contains a cosmological constant given by the vacuum energy density V . KSS showed, however, that for any V there exists a solution to (2) and (3) with the metric taking the form

$$ds^2 = e^{2A(y)}(-dt^2 + dx_1^2 + dx_2^2 + dx_3^2) + dy^2. \quad (4)$$

In other words, the 4-dimensional world is flat in spite of the presence of V . The dilaton field adjusts itself so that this solution exists. KSS referred to this as a self-tuning solution of the cosmological constant problem.

Arithmetically, this is possible because the equations of motion are solved separately for $y > 0$ and $y < 0$ and we have enough arbitrary integration constants that we can adjust in order to get the flat 4-dimensional world (4) we want. The equations of motion are, where $'$ denotes differentiation with respect to y ,

$$\frac{4}{9}(\varphi')^2 + A'' = -\frac{1}{6}V\delta(y), \quad (5)$$

$$(A')^2 - \frac{1}{9}(\varphi')^2 = 0, \quad (6)$$

and

$$\varphi'' + 4A'\varphi' = 0. \quad (7)$$

(5) is the difference between the $\mu\mu$ and 55 components of Einstein's equation, (6) the 55 component (which does not receive a brane contribution proportional to $\delta(y)$), and (7) the dilaton equation of motion. The 5-dimensional Bianchi identity states that only two of the three equations are independent. For example, (5) and (6) imply (7). Solving (6) for A' and inserting into (7) we see immediately that φ is given by the logarithm of y .

Thus, the solution may be chosen to be, for $y_+ > y > 0$

$$\varphi(y) = -\frac{3}{4} \log(y_+ - y) + d_+ \quad (8)$$

and

$$A(y) = \frac{1}{4} \log(y_+ - y) + a_+, \quad (9)$$

and for $y_- < y < 0$

$$\varphi(y) = \frac{3}{4} \log(y + y_-) + d_- \quad (10)$$

and

$$A(y) = \frac{1}{4} \log(y + y_-) + a_- \quad (11)$$

Here a_+, a_-, d_+, d_-, y_+ and y_- are integration constants. The continuity of φ and A at $y = 0$ determines d_+ and a_+ in terms of the other constants. Integrating (5) and (7) across $y = 0$ gives the jump conditions that

$$A'(y = 0^+) - A'(y = 0^-) = \frac{1}{4} \left(-\frac{1}{y_+} - \frac{1}{y_-} \right) = -\frac{1}{6} V \quad (12)$$

and

$$\varphi'(y = 0^+) - \varphi'(y = 0^-) = \frac{3}{4} \left(\frac{1}{y_+} - \frac{1}{y_-} \right) = 0 \quad (13)$$

These two equations merely fix y_+ and y_- in terms of V . Thus, KSS obtained a flat space solution, type I in their classification, for any V .

One can say that, in some sense, the cosmological constant problem is solved because we have a lot of integration constants. Of course, the framework of embedding our universe in a larger universe is of crucial importance.

The heavy price that one pays in the KSS solution is the appearance of naked singularities at y_+ and y_- . Near y_+ for example, the metric components $g_{00} = g_{ii} = e^{2A(y)}$ vanish like $\sqrt{y_+ - y}$. Various curvature invariants, for example the Ricci scalar $R = -20(A')^2 - 8A''$

diverge. These singularities are not clothed by an event horizon. That some types of naked singularity are not acceptable is known as cosmic censorship. Recently, Gubser [13] has studied in detail various singularities.

For metrics of the form $ds^2 = e^{2A(y)} g_{\mu\nu}(x) dx^\mu dx^\nu + dy^2$ we can see easily that $\sqrt{-G}R = e^{2A(y)} \sqrt{-g} R_{(4)} + \dots$ where the 4-dimensional scalar curvature $R_{(4)}$ is constructed out of $g_{\mu\nu}(x)$. Thus, the effective 4-dimensional Planck mass squared is proportional to $\int dy e^{2A(y)}$. In order to have a finite 4-dimensional Planck mass, KSS were forced to choose y_+ and y_- positive. They (rather arbitrarily) postulated that the universe is cutoff at y_+ and y_- and thus obtained a finite Planck mass squared proportional to

$$\int dy e^{2A(y)} = \int_0^{y_+} dy e^{2A(y)} + \int_{y_-}^0 dy e^{2A(y)}. \quad (14)$$

Once these integrals are thus cutoff to give finite values, we can obtain any desired value for the 4-dimensional Planck mass squared simply by shifting the additive integration constant allowed by (9) and (11) in the solution for $A(y)$.

Notice that while changing V does not appear to affect our 4-dimensional universe, which remains flat whatever the value of V , it does affect the 5-dimensional universe. In particular, it moves the naked singularities around.

This completes a necessarily brief review of KSS to which we refer for further details. We should perhaps mention here that KSS showed that more generally we can write $f(\varphi)$ instead of V and the same conclusion continues to hold. This is easy to see: in (5) V is replaced by $f(\varphi)$ and in (7) the term $+\frac{3}{8}\delta(y)\frac{\partial f}{\partial \varphi}$ is added to the right hand side. A particularly popular choice is $f(\varphi) = Ve^{b\varphi}$ as inspired by string theory.

At this point, it may be useful to say a few words about fine tuning and self tuning. Traditionally, one is to write down in a field theory Lagrangian all terms with dimension less than or equal to 4 allowed by symmetry. The coefficients of all such terms are to be regarded as free parameters. If we arbitrarily set one coefficient equal to another or to zero without being able to invoke a symmetry principle, then we are said to have fine tuned. We refer to this as the strong definition of fine tuning. Here we are dealing with

a non-renormalizable 5-dimensional action (1) which nobody would regard as fundamental. Presumably, this action is to be regarded as the low energy effective action of some more fundamental theory such as string theory. A traditionalist would say that KSS, starting with the action (1), has already committed fine tuning according to the strong definition by excluding all sorts of possible terms from (1). We feel that it is necessary to formulate a weak definition of fine tuning, according to which one can exclude or include any sorts of terms from the higher dimensional action without being accused of fine tuning. Thus, we would regard the KSS solution as not a fine tuning solution. Rather, KSS described their solution as self tuning, in the sense that the 4-dimensional world remains flat regardless of the value of V .

For example, the 5-dimensional action (1) could perfectly well contain the term $-\Lambda e^{a\varphi}$ corresponding to a bulk cosmological constant Λ , which KSS set to 0 in their self-tuning solutions. A traditionalist using the strong definition of fine tuning would definitely call setting Λ to 0 fine tuning; in the language of this subject, however, this is not called fine tuning. If Λ is not equal to 0, then as KSS themselves showed, the self tuning feature of their solution is lost. To obtain a flat 4-dimensional world, one has to adjust V to have a value determined by Λ . KSS called this fine tuning, as we think anybody would.

II. ACTION WITH GAUSS-BONNET TERM

We ask whether we could avoid naked singularities by adding higher derivative terms such as R^2 to the Einstein-Hilbert action. At the same time, of course we still have the highly non-trivial constraint that $\int_0^\infty dy e^{2A(y)}$ and $\int_{-\infty}^0 dy e^{2A(y)}$ have to be finite.

We replace the bulk action in (1) by

$$S_{bulk} = \int d^5x \sqrt{-G} \left[R + a R^2 - 4b R^{MN} R_{MN} + c R^{MNPQ} R_{MNPQ} - \frac{4}{3} (\nabla\varphi)^2 - \mathcal{V}(\varphi) \right], \quad (15)$$

We have also included a potential $\mathcal{V}(\varphi)$ for the dilaton field.

We will proceed in stages. We will first study the effect of the higher derivative terms with $\mathcal{V}(\varphi) = 0$. Then in section III we will include $\mathcal{V}(\varphi)$.

Varying S_{bulk} with respect to the metric tensor we obtain the equation of motion

$$\begin{aligned}
& \left\{ R_{MN} - \frac{1}{2} G_{MN} R - \frac{1}{2} G_{MN} \left[a R^2 - 4b R^{PQ} R_{PQ} + c R^{PQST} R_{PQST} \right] \right. \\
& + 2a R R_{MN} - 4c R_{MP} R_N^P + 2c R_{MPQS} R_N^{PQS} + 4(2b - c) R^{PQ} R_{MPQN} \\
& \left. + 2(a - b) G_{MN} R_{;P}^P - 2(a - 2b + c) R_{;M;N} - 4(b - c) R_{MN}{}^{;P}{}_{;P} \right\} \\
& = T_{MN},
\end{aligned} \tag{16}$$

where

$$T_{MN} = \frac{4}{3} \left[\nabla_M \varphi \nabla_N \varphi - \frac{1}{2} G_{MN} (\nabla \varphi)^2 \right] - \frac{1}{2} G_{MN} \mathcal{V}(\varphi). \tag{17}$$

These higher derivative terms naturally arise in the low energy effective action of string theory [14,15]. For our purposes, we do not inquire of their deeper origin but simply treat the action as “phenomenological.” (For $a = b = 0$, (16) agrees with [15].)

We will choose $a = b = c = \lambda$ so that the added term $\lambda(R^2 - 4R^{MN} R_{MN} + R^{MNPQ} R_{MNPQ})$ is of the Gauss-Bonnet form. One sees from the equation of motion that all terms involving fourth derivative vanish in this case and (16) reduces to the result given in [16]. As is well known, the Gauss-Bonnet combination is a topological invariant in 4-dimensional spacetime. In higher dimensional spacetime, it is not a topological invariant but nevertheless has particularly attractive properties, as discovered by Zwiebach [17] and explained by Zumino [18]. We proceed in an exploratory spirit we believe appropriate for this stage of development of this nascent subject and do not apologize further for this specific choice.

With the ansatz for the metric in (4), we obtain

$$\frac{4}{9}(\varphi')^2 + [1 - 4\lambda(A')^2] A'' = -\frac{1}{6}f(\varphi) \delta(y), \tag{18}$$

$$(A')^2 - \frac{1}{9}(\varphi')^2 - 2\lambda(A')^4 = 0, \tag{19}$$

and

$$\varphi'' + 4A'\varphi' = \frac{3}{8} \delta(y) \frac{\partial f}{\partial \varphi}(\varphi). \tag{20}$$

The matching condition at the location of the 3-brane, $y = 0$, becomes

$$\frac{8}{3} \varphi'(y) \Big|_{0^-}^{0^+} = \frac{\partial f}{\partial \varphi}(\varphi(0)), \quad (21)$$

$$-6 \left[A'(y) - \frac{4}{3} \lambda A'(y)^3 \right] \Big|_{0^-}^{0^+} = f(\varphi(0)), \quad (22)$$

and the continuity condition for $\varphi(y)$ and $A(y)$ at $y = 0$. In the limit $\lambda \rightarrow 0$ these equations reduce to the ones studied in KSS of course. Again, the Bianchi identity assures us that only two out of three bulk equations are independent. After solving two equations we can use the third one as a convenient check.

We solve these equations in the bulk, for $y > 0$ say. The scalar equation (20) gives

$$\varphi' = d e^{-4A(y)} \quad (23)$$

with d an integration constant. Inserting this into (19) we obtain

$$(A')^2 = \frac{1}{4\lambda} \left(1 \pm \sqrt{1 - \frac{8d^2}{9} \lambda e^{-8A}} \right) \quad (24)$$

From (24) we see immediately that, for $\lambda > 0$, we must have $e^{8A} > (8d^2/9) \lambda$ and hence the 4-D Planck mass squared is infinite unless spacetime is cut off by some singularity. To solve (24), we change variable to $\kappa(y) = e^{-4A} = 1/G_{tt}^2$ and (24) becomes

$$(\kappa')^2 = \frac{4\kappa^2}{\lambda} \left(1 \pm \sqrt{1 - \frac{8d^2}{9} \lambda \kappa^2} \right), \quad (25)$$

which can be readily solved.

For $\lambda > 0$, we obtain

$$y(\kappa) = y_0 \pm \frac{1}{\sqrt{8\lambda}} \left[\log \left(\cot \frac{\theta + \pi}{4} \right) - \csc \frac{\theta}{2} \right], \quad (26)$$

$$y(\kappa) = y_0 \pm \frac{1}{\sqrt{8\lambda}} \left[\log \left(\tan \frac{\theta}{4} \right) - \sec \frac{\theta}{2} \right], \quad (27)$$

where

$$\sin \theta = \frac{2d}{3} \sqrt{2\lambda} \kappa = \frac{2d}{3} \sqrt{2\lambda} \frac{1}{G_{tt}^2}, \quad (28)$$

$-\pi/2 \leq \theta \leq \pi/2$ and y_0 is an integration constant. These four solutions correspond to the four different choices of signs in (25). In order to have finite 4-D Planck mass, there are again naked singularities in the fifth dimension.

For $\lambda = -|\lambda| < 0$, only the plus sign in (25) is allowed and we have

$$y(\kappa) = y_0 \pm \frac{1}{\sqrt{8|\lambda|}} \left[2 \tan^{-1} \left(\tanh \frac{\psi}{4} \right) - \operatorname{csch} \left(\frac{\psi}{2} \right) \right], \quad (29)$$

where

$$\sinh \psi = \frac{2d}{3} \sqrt{2|\lambda|} \kappa. \quad (30)$$

In this case, the naked singularity persists if we demand finite 4-D Planck mass. In conclusion, while we still have the nice self tuning solution of the cosmological constant problem as in KSS we are also stuck with the naked singularities.

III. ADDING A DILATON POTENTIAL

We next explore what happens if in the action we add a potential $\mathcal{V}(\varphi)$ for the scalar field. A potential can arise in string theory from higher order corrections, but again in the “phenomenological” spirit of this paper we do not concern ourselves with its origin. The equations of motion are modified to

$$\frac{4}{9}(\varphi')^2 + [1 - 4\lambda(A')^2] A'' = -\frac{1}{6}f(\varphi) \delta(y), \quad (31)$$

$$(A')^2 - \frac{1}{9}(\varphi')^2 - 2\lambda(A')^4 = -\frac{1}{12} \mathcal{V}(\varphi), \quad (32)$$

and

$$\varphi'' + 4A'\varphi' = \frac{3}{8} \frac{\partial \mathcal{V}(\varphi)}{\partial \varphi} + \frac{3}{8} \delta(y) \frac{\partial f}{\partial \varphi}(\varphi) \quad (33)$$

When $\lambda = 0$, these equations have been studied previously and a solution-generating method inspired by supergravity was suggested in [11,19,20]. It is shown that, for $\lambda = 0$, if $\mathcal{V}(\varphi)$ takes the special form

$$\mathcal{V}(\varphi) = \frac{27}{4} \left(\frac{\partial W(\varphi)}{\partial \varphi} \right)^2 - 12 W(\varphi)^2, \quad (34)$$

then a solution to

$$\varphi' = \frac{9}{4} \frac{\partial W(\varphi)}{\partial \varphi} \quad (35)$$

$$A' = -W(\varphi), \quad (36)$$

is also a solution to the equations of motion. Furthermore, by counting the number of integration constants one can show that the solution space of (34)-(36) coincides with the solution space of the equations of motion following from (31)-(33) for $\lambda = 0$ [11]. $W(\varphi)$ is sometimes called the “superpotential” for obvious reason, though no supersymmetry is involved here. Note that in this case

$$A'' = -\frac{9}{4} \left(\frac{\partial W(\varphi)}{\partial \varphi} \right)^2 \leq 0. \quad (37)$$

From (31) and (32), staying in the bulk and thus ignoring $\delta(y)$ for now, we obtain similar first order equations for nonzero λ :

$$\mathcal{V}(\varphi) = \left[\frac{27}{4} \left(\frac{\partial W(\varphi)}{\partial \varphi} \right)^2 + \frac{3}{2\lambda} \right] (1 - 4\lambda W(\varphi)^2)^2 - \frac{3}{2\lambda}, \quad (38)$$

$$A' = -W(\varphi), \quad (39)$$

$$\varphi' = \frac{9}{4} (1 - 4\lambda W(\varphi)^2) \frac{\partial W(\varphi)}{\partial \varphi}. \quad (40)$$

In terms of the superpotential $W(\varphi)$, the matching condition at the location of the 3-brane, $y = 0$, is now

$$6 \frac{\partial}{\partial \varphi} \left[W(\varphi(y)) - \frac{4}{3} \lambda W(\varphi(y))^3 \right] \Big|_{y=0^-}^{y=0^+} = \frac{\partial f}{\partial \varphi}(\varphi(0)), \quad (41)$$

$$6 \left[W(\varphi(y)) - \frac{4}{3} \lambda W(\varphi(y))^3 \right] \Big|_{y=0^-}^{y=0^+} = f(\varphi(0)). \quad (42)$$

If, in a specific model, the 3-brane tension $f(\varphi)$ is given by $12(W - \frac{4}{3} \lambda W^3)$, these jump conditions can be satisfied identically.

It is simple to check that (33) is satisfied automatically. Generalization to n scalar fields $\varphi = (\varphi_1, \dots, \varphi_n)$ is achieved by the following replacement:

$$W(\varphi) \rightarrow W(\boldsymbol{\varphi}), \quad (43)$$

$$\left(\frac{\partial W(\varphi)}{\partial \varphi}\right)^2 \rightarrow \frac{\partial W(\boldsymbol{\varphi})}{\partial \boldsymbol{\varphi}} \cdot \frac{\partial W(\boldsymbol{\varphi})}{\partial \boldsymbol{\varphi}}. \quad (44)$$

In the limit $\lambda \rightarrow 0$, these equations reduces to (34)-(36). Note that the dilaton potential $\mathcal{V}(\varphi)$ is now bounded below if λ is positive. The second derivative of A now becomes

$$A'' = -\frac{9}{4} \left(\frac{\partial W(\varphi)}{\partial \varphi}\right)^2 (1 - 4\lambda W(\varphi)^2) \quad (45)$$

and is no longer to be non-positive always. In five dimensional gauged supergravity, (35) and (36) arise as conditions for unbroken supersymmetry, and (37) is used to prove a c -theorem [21]. It would be interesting to study the implications of (38)-(40) in the context of these models.

The reason that we are able to obtain the first order Bogomol'nyi equations in the presence of higher derivative terms is that we have chosen the particularly nice Gauss-Bonnet combination. For our purposes we regard the method as simply a method for solving coupled differential equations. Thus, for a given choice of W we generate a solution for some $\mathcal{V}(\varphi)$. Note that if we did not use the Gauss-Bonnet combination we would have third and fourth derivatives of A appearing in (31).

We now choose $W(\varphi) = s\varphi$ to be a linear function of φ . Note that s has dimension of an inverse length. We find it convenient to define the length scale $l = 2/(9\sqrt{\lambda}s^2)$ and the dimensionless ratio $\sigma = (l/\sqrt{\lambda})^{\frac{1}{2}}$.

Following the steps outlined above, we generate the solution for $y > 0$

$$\varphi(y) = \frac{3}{2\sqrt{2}} \sigma \tanh\left(\frac{y - y_+}{l}\right), \quad (46)$$

$$A(y) = -\frac{1}{2} \sigma^2 \log \left[\cosh\left(\frac{y - y_+}{l}\right) \right] + k_+ \quad (47)$$

and for $y < 0$

$$\varphi(y) = \varepsilon \frac{3}{2\sqrt{2}} \sigma \tanh\left(\frac{y - y_-}{l}\right), \quad (48)$$

$$A(y) = -\frac{1}{2} \sigma^2 \log \left[\cosh \left(\frac{y - y_-}{l} \right) \right] + k_- \quad (49)$$

The potential is a familiar double well

$$\mathcal{V}(\varphi) = \frac{3}{2\lambda} \left(\frac{1}{\sigma^2} + 1 \right) \left(1 - \frac{8}{9} \frac{\varphi^2}{\sigma^2} \right)^2 - \frac{3}{2\lambda}, \quad (50)$$

Note from (33) that we have freedom in choosing the sign of φ represented by $\varepsilon = \pm 1$ in (48). The φ solution we have is a kink in the fifth dimension interpolating two vacua of the potential. The spacetime is asymptotically AdS and there is no singularity at all. Moreover, the 4-D Planck mass is finite.

Unfortunately, in this particular example, the self tuning feature of KSS is also lost. To see this, take $f(\varphi) = V$ for simplicity. The continuity of φ and φ' fixes $\varepsilon = +1$ and $y_+ = y_- \equiv y_*$. The continuity of A fixes $k_+ = k_-$ while (21) tells us that the jump in $[1 - \frac{4}{3}\lambda(A')^2]A'$ across $y = 0$ is equal to $-\frac{1}{6}V$. But this cannot be if V is not zero since $A(y) = -\frac{1}{2}\sigma^2 \log \cosh \frac{1}{l}(y - y_*)$ is perfectly smooth across $y = 0$. The crucial point here is that we no longer have the freedom of including an additive constant in the solution for φ in this example.

We can also choose a more general superpotential of the form $W(\varphi) = s\varphi + r$. We find that, by choosing $\lambda < 0$ and $s = \sqrt{2/(9|\lambda|)}$, the dilaton potential is a constant

$$\mathcal{V}(\varphi) = \frac{3}{2|\lambda|} \quad (51)$$

which acts like a bulk cosmological constant and is independent of r in the superpotential $W(\varphi)$. Thus r plays the role of an integration constant and we recover the self tuning feature in KSS. Unfortunately, with negative λ the hyperbolic functions in (46)-(49) turn into trigonometric functions and we again have naked singularities in the bulk.

In these two examples we constructed, we need either fine tuning to avoid the naked singularities or naked singularities to maintain the self tuning feature.

NOTE ADDED

After this paper was submitted a paper [22] appeared in which the authors proved a no-go theorem which states that, in the scenario of [2,3], one needs either fine-tuning or naked singularities to achieve the flatness of our universe. Although the two examples we constructed here, in the presence of Gauss-Bonnet term, are consistent with this no-go theorem, we would like to point out that a crucial ingredient of the proof given in [22] , $A'' \leq 0$, is not true in our case, as can be seen from (45). Therefore there might still be hope of retaining self-tuning feature without invoking naked singularities.

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